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UNITED STATES NAVAL POSTGRADUATE SCHOOL

NUMERICAL COMPUTATION OF IMAGE SIZE AND DISTANCE FOR CONVERGING CHARGED PARTICLE BEAMS

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TECHNICAL REPORT/RESEARCH PAPER No. 55

NUMERICAL COMPUTATION OF IMAGE SIZE AND DISTANCE FOR CONVERGING CHARGED PARTICLE BEAMS

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UNITED STATES NAVAL POSTGRADUATE SCHOOL

Monterey, California

Rear Admiral E. J. O'Donnell, USN, Superintendent

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ABSTRACT:

In this paper are presented the results of numerical computations of the effect of space charge upon the image size and image distance in a converging beam of charged particles. The minimum beam radius (image size) and its position (image distance) are plotted as a function of beam perveance ranging from 10^{-13} to 10^{-6}. The beam convergence angle was varied from about 0.03 to 0.15 radians and calculations were carried out for 3 values of aperture radius (0.1 cm, 0.25 cm and 0.50 cm). The results are given for electrons, $^{1}H^{+}$, $^{7}Li^{+}$, $^{20}Ne^{+}$, and $^{85}Rb^{+}$.

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I. INTRODUCTION

In recent years there has been an increasing demand for low energy ion beams, well defined in space and energy, to be used in experimental investigations of atomic collision processes. In the energy region below 100 eV the effects of space charge upon usable beam intensity, beam shape and ion energy become so serious as to represent the limiting factors in many experiments.

Pierce[1] has designed a widely used lens utilizing curved electrodes to create the fields required to simulate flow between concentric spheres. Although the Pierce gun has been used for the production of low energy ion beams with some success, it was recently shown by Simpson and Kuyatt[2] that there exists a fundamental limitation on the beam intensity which can be obtained by any unipotential gun operating at low energies. They show that in order to attain the current densities required for saturation, in the presence of thermal effects at the emitter surface, one would have to exceed the space charge limitations imposed by the diode nature of any unipotential gun. From this it is concluded that the range of applicability of unipotential guns lies at comparatively high energies and wide beam convergence angles. To operate outside these limits it is necessary to decouple the thermal energy and space charge effects by first accelerating the particles to a fairly high energy and then decelerating them to their final low energy. Simpson and Kuyatt[3] have designed an electron gun using this multistaging principle and we have adapted their design for use with low energy ion beams.

A schematic diagram of the gun is given in Fig. 1. The first stage extracts the particles at high energies and forms a beam which is then decelerated to the desired final energy in the second stage. The first, or extraction stage is based on designs given by Soa[4] and the deceleration stage is based on the properties of plane electrodes, as given by Spangenberg and Field.[5] The electrode potentials V_g and V_a are measured with respect to the emitter which is maintained at the desired

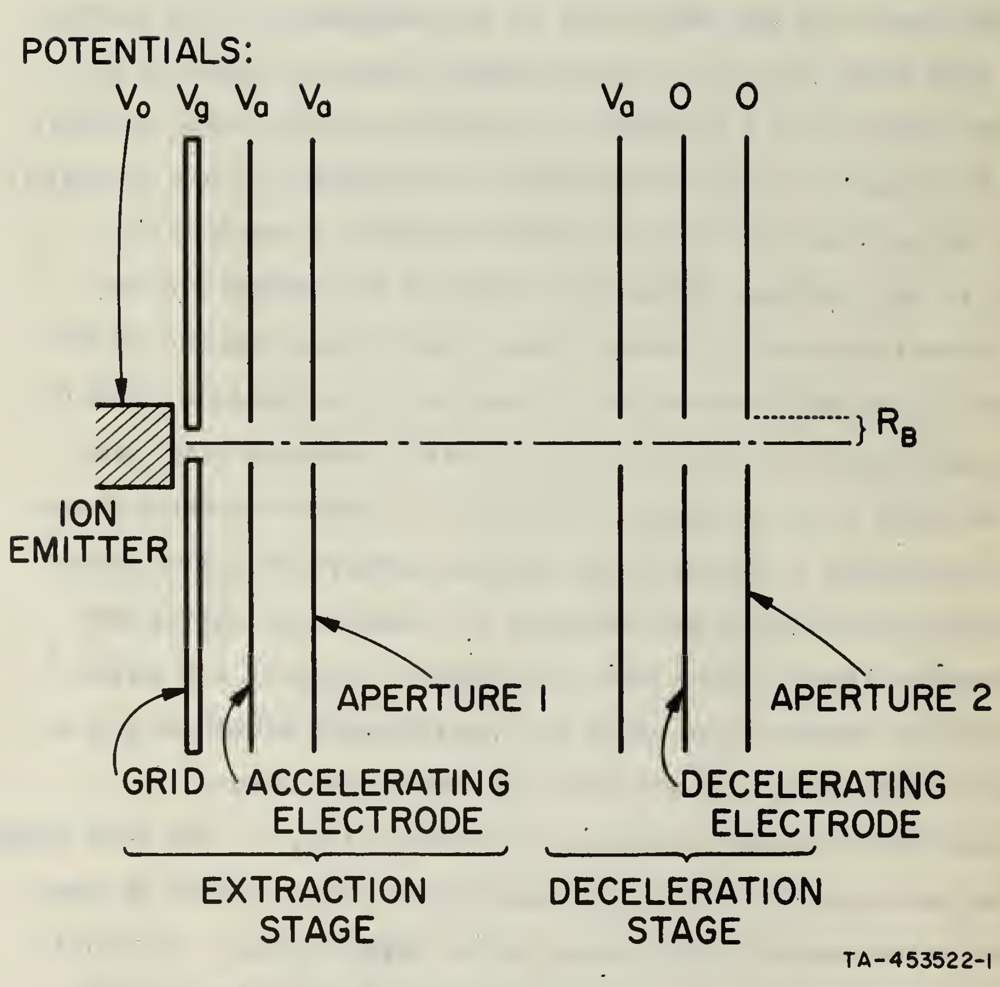
POTENTIALS:
V_o V_g V_a V_a
V_a O O
R_B
ION EMITTER
APERTURE I
APERTURE 2
GRID
ACCELERATING ELECTRODE
DECELERATING ELECTRODE
EXTRACTION STAGE
DECELERATION STAGE
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FIGURE 1

beam potential, V_o , while the exit electrodes are grounded.

In addition to circumventing the basic problem described above, this design has considerable mechanical advantages over the conventional Pierce gun. All electrodes are plane; displacement of the deceleration stage with respect to the extraction stage and variation of the electrode potentials allows one to obtain a wide variety of beam shapes, intensities and energies quickly and conveniently.

The details and results of our experimental investigation of this gun are reported elsewhere[6] and will only be summarized very briefly here. To obtain a low energy ion beam we used a thermal emitter ($Li_2O \cdot Al_2O_3 \cdot n\ SiO_2$) as a source of Li^+ ions[7]. Current densities of 10^{-8} to 10^{-6} amps/cm^2 at beam energies of 2 to 100 eV were obtained and both the spatial and energy distributions of the ions were investigated. Minimum beam radii of 1 to 2 mm occurred a few centimeters beyond the last element of the gun. Beam profiles were measured as a function of axial distance and well collimated beams of up to 16cm were obtained. The energy spread in the beam was found to be about 0.22 eV in good agreement with the expected thermal energy spread from the source which operates at about 1200°C. Mass analysis shows that beam purities are well in excess of 99% $^7Li^+$ when isotopically enriched emitter material is used.

II. BEAM DYNAMICS

The beam geometry in the absence of space charge effects is shown in Fig. 2. As the beam leaves the aperture of radius R_B the rays emanating from the edge are initially directed as shown and those from interior points are contained within the outer rays so as to be focused toward a spot of radius r_a a distance L away. The origin of coordinates is such that the beam travels along the z axis, the coordinates of the aperture is z_B and of the image z_a . The beam is assumed to be axially symmetric.

In considering the space charge effects we shall follow the work of Glaser[8] and the analyses of Watson[9] and Wendt[10]. We shall assume

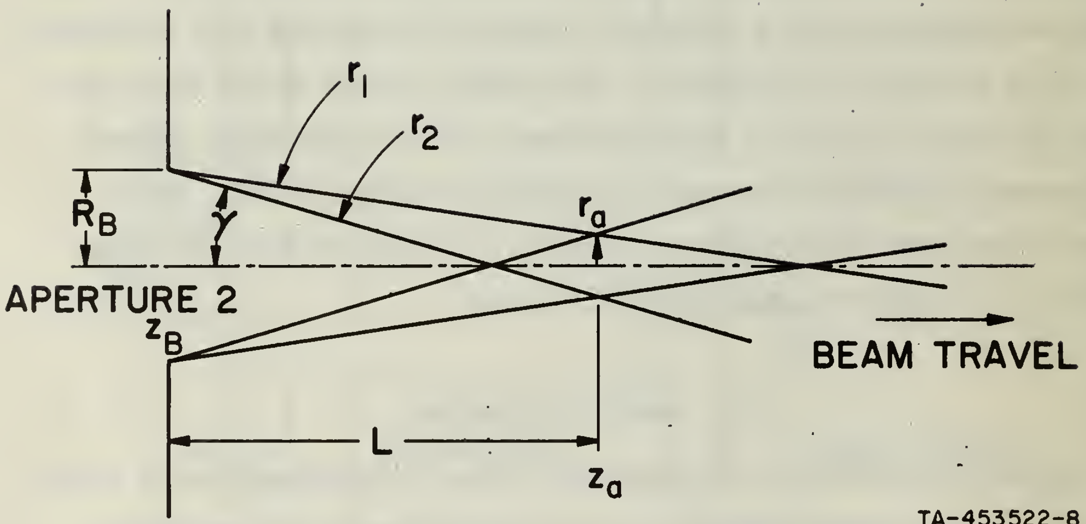


FIGURE 2

a beam of charged particles of sufficient density to be considered a continuous medium for purposes of constructing energy functions.

The effect of the space charge of the beam is to deflect the rays away from the axis. We assume that the current density is uniform across the beam cross section and zero outside the outer rays, and that the total current is constant. If the convergence of the rays is sufficiently small then a small segment of the beam forms a cylinder of nearly constant radius in which the axial forces are nearly zero, and hence the axial velocity $\dot{z}$ is very nearly constant and given by

$$\dot{z} = \sqrt{\frac{2eV}{m}} \tag{1}$$

where e = charge on the particle, V = beam potential, and m = particle mass.

We shall now proceed to calculate the trajectories of the outer and inner rays as shown in Fig. 2 under the influence of space charge (in field free space) subject to the simplifying assumptions given above.

Trajectory of the Outer Ray

The radial component of the electric field due to the space charge can be written

$$E_{r_1} = \frac{i}{2\pi\varepsilon_0 r_1}\sqrt{\frac{m}{2eV}} \tag{2}$$

where i is the total current, eV is the beam energy, and r_1 is the radius of the outer ray at any point. Let

$$\eta = \frac{e}{m}$$

be the charge to mass ratio of the particles. The radial equation of motion of a particle on the outer ray is then

$$m\ddot{r}_1 = \left(\frac{ei}{2\pi\varepsilon_0 r_1}\right)\frac{1}{\sqrt{2\eta V}} \tag{3}$$

If we define the constant K_1 to be

$$K_1 = \frac{i}{2\pi\varepsilon_0\sqrt{2\eta V}} \tag{4}$$

we may write Eq. (3) as

$$\ddot{r}_1 = K_1\frac{\eta}{r_1} \tag{5}$$

Multiplying both sides by dr_1 and integrating gives

$$\frac{1}{2}\dot{r}_1^2 = K_1 \eta \ln\left(\frac{r_1}{r_0}\right) \tag{6}$$

where the constant of integration r_o is chosen such that at

$$r_1 = r_0 \ , \ \dot{r}_1 = 0$$

Using equations (1) and (4) we can write

$$\dot{r}_1 = -\dot{z}\left[\frac{K_1}{V}\ln\left(\frac{r_1}{r_0}\right)\right]^{1/2} \tag{7}$$

where we choose the negative root in order to restrict consideration to the portion of the beam between the aperture and the focal point, i.e., the region where the beam is convergent and $\dot{r}_1 < 0$. This gives on integration

$$z - z_B = -\int_{R_B}^{r_1}\left[\frac{K_1}{V}\ln\left(\frac{r_1'}{r_0}\right)\right]^{-1/2} dr_1'$$

where the primes denote variables of integration.

Finally, setting

$$s = \left[\ln\left(\frac{r_1}{r_0}\right)\right]^{1/2} \tag{8}$$

we find

$$z - z_B = -2K\int_{s_B}^{s} \exp\{s'^2\}\, ds' \tag{9}$$

where

$$K = r_0\sqrt{\frac{V}{K_1}}$$

If we define the Dawson function

$$D(s) = \int_0^s \exp\{s'^2\}\, ds' \tag{10}$$

then Eq. (9) becomes

$$z - z_B = -2K\left[D(s) - D(s_B)\right] \tag{11}$$

Since tables of D(S) are available[11] we can evaluate equation 11 in terms of the constant K. This equation gives r_1 as an implicit function of z and the parameters R_B, r_a, L, η and the beam perveance P.

To express K in terms of the minimum radius r_o and the beam perveance $P = \frac{i}{V^{3/2}}$ we note from above, that

$$K = r_0\sqrt{\frac{V}{K_1}} \tag{12}$$

where $$K_1 = \frac{i}{2\pi\varepsilon_0 \sqrt{2\eta V}} \tag{4}$$

Thus $$K = r_0 \sqrt{\frac{2\pi\varepsilon_0 \sqrt{2\eta}\; V^{3/2}}{i}}$$

or $$K = C r_0 P^{-\frac{1}{2}} \qquad \text{where} \qquad C = \sqrt{2\pi\varepsilon_0 \sqrt{2\eta}} \tag{13}$$

Trajectory of the Inner Ray

We know from Gauss' Law that only the charge enclosed by the inner ray will affect its trajectory. Assuming a uniform radial charge distribution, the ratio of the current included within r_2 to the current included within r_1 will be $\frac{r_2^{\ 2}}{r_1^{\ 2}}$ and since the field depends only on the enclosed charge we can use Eqs. (2) and (4) to write

$$E_{r_2} = K_1 \frac{r_2}{r_1} \tag{14}$$

and the radial equation of motion of a particle on the inner ray is

$$\ddot{r}_2 = \eta K_1 \frac{r_2}{r_1^2} \tag{15}$$

where r_1 is given by

$$r_1 = r_0 \exp\{s^2\} \tag{16}$$

from Eq. (8).

We may differentiate r_2 parametrically as

$$\frac{dr_2}{dt} = \frac{dr_2}{ds}\frac{ds}{dr_1}\frac{dr_1}{dt} = \frac{dr_2}{ds}\frac{ds}{dr_1}\dot{r}_1$$

where $\dot{r}_1$ is given by (6). ds/dr_1 may be found from (8), giving

$$\dot{r}_2 = \left(\sqrt{2K_1\eta}\right)\frac{\exp\{-s^2\}}{2r_0}\frac{dr_2}{ds} \tag{17}$$

and

$$\ddot{r}_2 = K_1\eta \frac{\exp\{-2s^2\}}{2r_0^2}\left(\frac{d^2r_2}{ds^2} - 2s\frac{dr_2}{ds}\right) \tag{18}$$

Putting (18) in (15) gives

$$\frac{d^2 r_2}{ds^2} - 2s\frac{dr_2}{ds} - 2r_2 = 0 \qquad (19)$$

which has the solution[12]

$$r_2 = C_1 \exp\{s^2\}\left[1 + C_2\,\phi(s)\right] \qquad (20)$$

where C_1 and C_2 are constants of integration, and

$$\phi(s) = \frac{2}{\sqrt{\pi}}\int_0^s \exp\{-s'^2\}\,ds' \qquad (21)$$

ϕ (s) is the well-known error function which is extensively tabulated[11].

We evaluate the constants of integration by noting that at $z = z_B$ (see Fig. 2)

$$r_{1B} = r_{2B} = R_B$$

and

$$\left(\frac{dr_1}{dz}\right)_B = -\frac{R_B - r_a}{L}$$

$$\left(\frac{dr_2}{dz}\right)_B = -\frac{R_B + r_a}{L} \qquad (22)$$

Note also that from equation (9)

$$\frac{dz}{dr_1} = -2K\frac{d}{dr_1}\int_0^s \exp\{s'^2\}\,ds'$$

Applying Leibnitz' rule gives

$$\frac{dz}{dr_1} = -2K\frac{ds}{dr_1}\exp\{s^2\} = -\frac{K}{r_0 s}$$

So

$$\frac{dr_1}{dz} = -\frac{r_0 s}{K}$$

and

$$\left(\frac{dr_1}{dz}\right)_B = -\frac{r_0 s_B}{K} \qquad (23)$$

whence

$$s_B = \frac{K(R_B - r_a)}{r_0 L} \qquad (24)$$

From (16)

$$\frac{R_B}{r_0} = \exp\{s_B\}^2$$

so $$r_0 = R_B \exp\left\{-c^2\left(\frac{r_a}{R_B}-1\right)^2 \frac{R_B{}^2}{L^2 P}\right\} \tag{25}$$

Now from (20)

$$R_B = C_1 \exp\{S_B^2\}\left[1 + C_2 \phi(S_B)\right] \tag{26}$$

and

$$\frac{dr_2}{dz} = C_1 \exp\{s^2\} \frac{dr_1}{dz}\frac{ds}{dr_1}\left[2s(1+C_2\phi) + C_2\frac{d\phi}{ds}\right] \tag{27}$$

Applying Leibnitz'rule to (21) gives

$$\frac{d\phi}{ds} = \frac{2}{\sqrt{\pi}} \exp\{-s^2\} \tag{28}$$

and

$$\frac{ds}{dr_1} = \frac{1}{2 s r_1} \tag{29}$$

Putting (23), (28), (29), and the second equation of (22) into (27) and evaluating at $r_2 = R_B$, gives

$$\frac{R_B + r_a}{L} = \frac{C_1}{K}\left[S_B(1 + C_2\phi_B) + \frac{C_2}{\sqrt{\pi}}\exp\{-S_B^2\}\right] \tag{30}$$

where ϕ_B denotes $\phi(S_B)$. Solving (26) for C_1 and putting this and (24) in (30) gives

$$\frac{C_2}{1 + C_2\phi_B} = \frac{2\sqrt{\pi}\, K R_B r_a}{r_0^2 L}$$

Let

$$N = \frac{2\sqrt{\pi}\, K R_B r_a}{r_0^2 L} \tag{31}$$

With this we have immediately

$$C_2 = \frac{N}{1 - N\phi_B} \tag{32}$$

and from (26)

$$C_1 = r_0(1 - N\phi_B) \tag{33}$$

so the complete solution for r_2 (see equation 20) is given by

$$r_2 = r_0 \exp\{s^2\}\left[1 - N(\phi_B - \phi)\right] \tag{34}$$

where r_o is given by (25), N by (31), s_B by (24) and ϕ by (21). It should be noted that equation (34) gives r_2 as a function of the constants R_B, r_a, L, η and the beam perveance P at every value of z.

Calculation of Minimum Beam Radius and Postion

The spreading of the rays under the influence of space charge is illustrated in Fig. 3. We shall distinguish between two cases depending on whether the value of the beam perveance P is larger than or smaller than a critical perveance P_{cr} defined below:

Case 1; $P > P_{cr}$ (Fig. 3a): For this case the outer ray reaches its minimum before its intersection with the inner ray originating at the diametrically opposite side of the aperture. Thus the outer ray reaches its minimum value $r_1 = r_o$ before $-r_2 = r_1$ and hence the minimum radius occurs at $z = z_o$.

Let $z_o - z_B = L_F$. At $r_1 = r_o$, $s = 0$ (from equation 8). Since $D(o) = 0$ we get from equation (11)

$$L_F = 2K\,D(s_B) \tag{35}$$

and $r_o = r_F$ is given by equation (25).

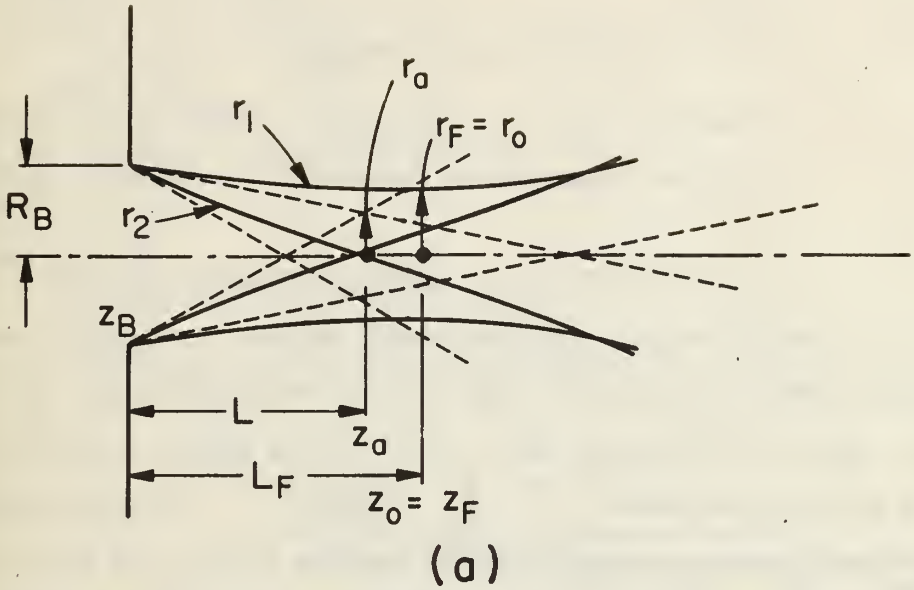
Case 2; $P < P_{cr}$ (Fig. 3b): Here the outer ray will be intersected by the inner ray originating at the diametrically opposite side of the aperture, before r_1 reaches its minimum value r_o. If we call this point of intersection z_F then at $z = z_F$, $-r_2 = r_1 = r_F$.

Let $z_F - z_B = L_F$. The minimum radius occurs where $r_1 = -r_2$. Equation (16) and (34) gives

$$1 - N(\phi_B - \phi_F) = -1$$

from which

$$\phi_F = \phi_B - \frac{2}{N} \tag{36}$$



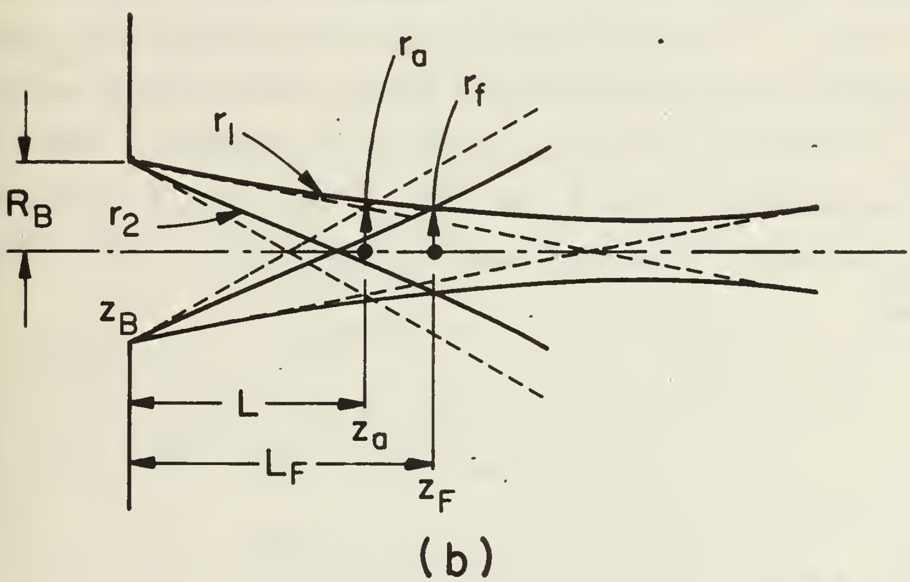


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FIGURE 3

s_F can be found from (36) and we find r_F from

$$r_F = r_o \exp \left\{ s_F^2 \right\} \tag{37}$$

and

$$L_F = 2K \left[D(s_B) - D(s_F) \right] \tag{38}$$

When $P = P_{cr}$, L_F (case 1) = L_F (case 2) and the method of either case may be employed in evaluating r_F and L_F.

III. RESULTS OF NUMERICAL COMPUTATIONS

In this section we present the results of numerical computations for the image distance L_F (Equ. 35 and 38) and the image radius r_F (Equ. 25 and 37). Each of the figures (Fig. 4-18) shows 3 curves of L_F as a function of beam perveance $P \left(= \frac{i}{V^{3/2}} \right)$ for three values of the beam convergence angle γ (Fig. 2) ($\tan \gamma = 0.03$, 0.09 and 0.15). On the same figure we plot r_F as a function of P for the same three values of $\tan \gamma$. The curves are aligned so that values of L_F and r_F can be read for any given value of P and the same value of $\tan \gamma$ by drawing a vertical line at the proper value of perveance. Three values of the final aperture radius R_B (See Fig. 2) ($R_B = 0.1$, 0.25 and 0.50 cm) and five mass values were used: electrons, $^{1}H^{+}$, $^{7}Li^{+}$, $^{20}Ne^{+}$ and $^{85}Rb^{+}$.

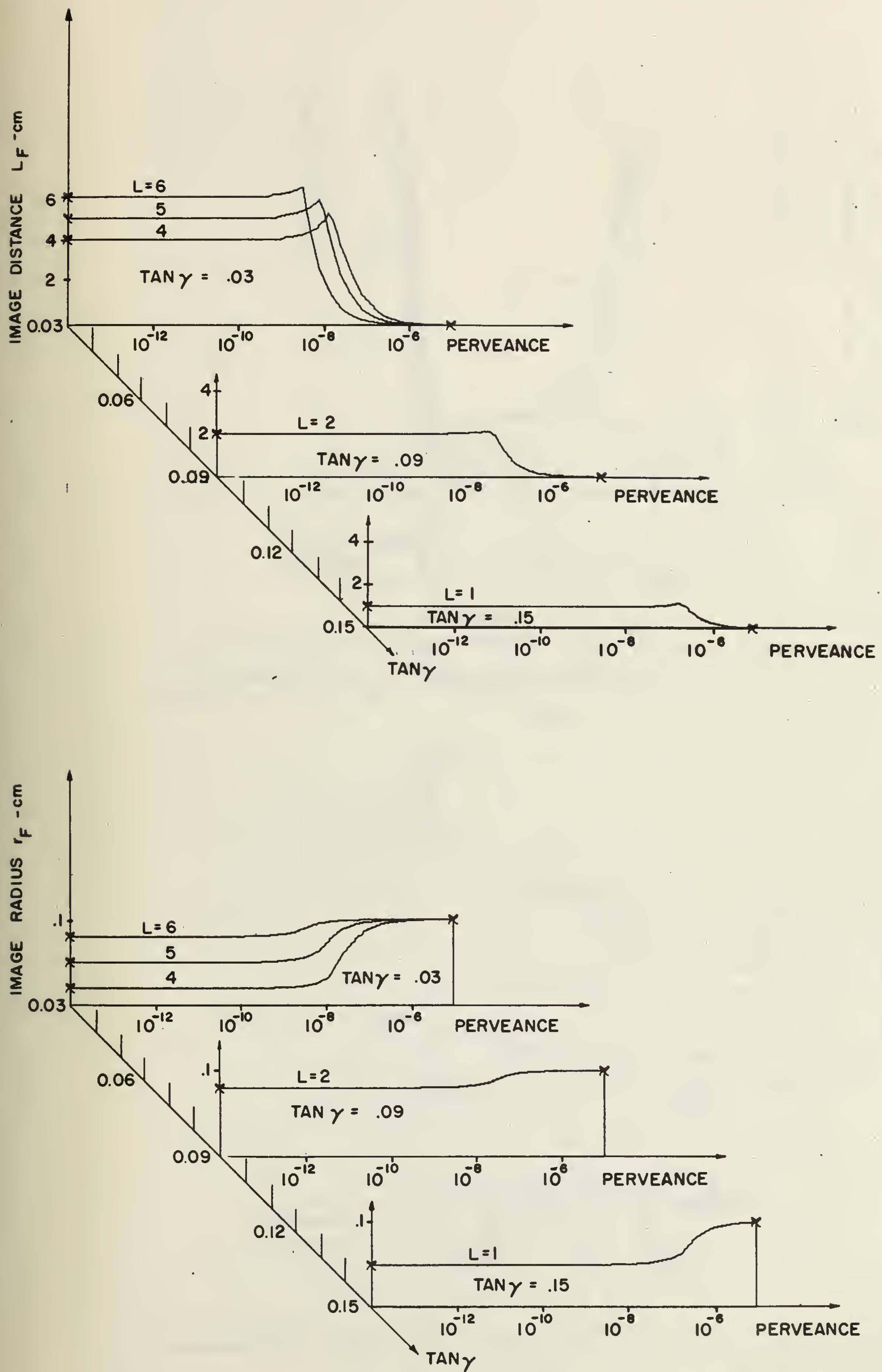


IMAGE DISTANCE L_F AND IMAGE RADIUS r_F FOR ELECTRON BEAM (R =0.10cm)

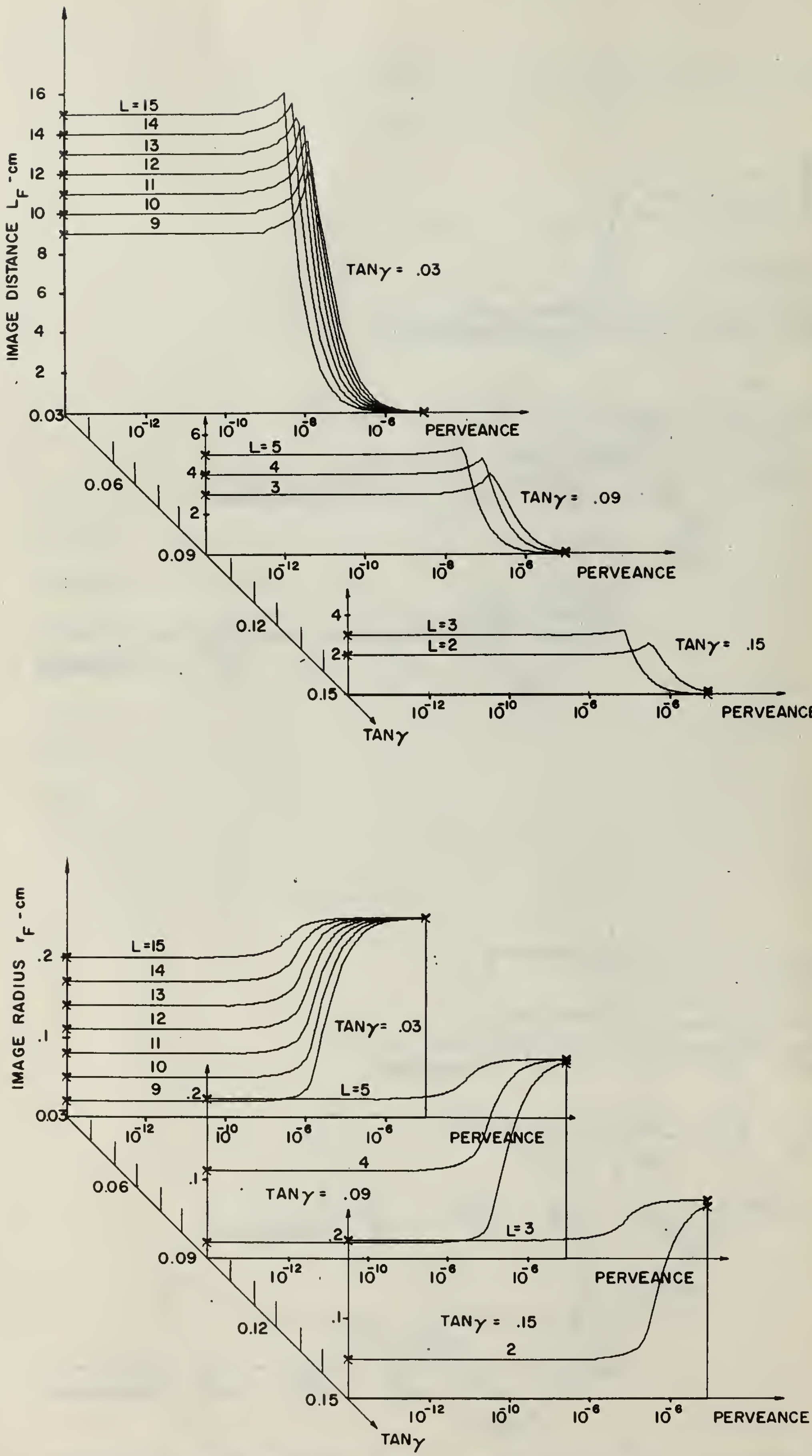


IMAGE DISTANCE L_F AND IMAGE RADIUS r_F FOR ELECTRON BEAM (R_B=0.25 cm)

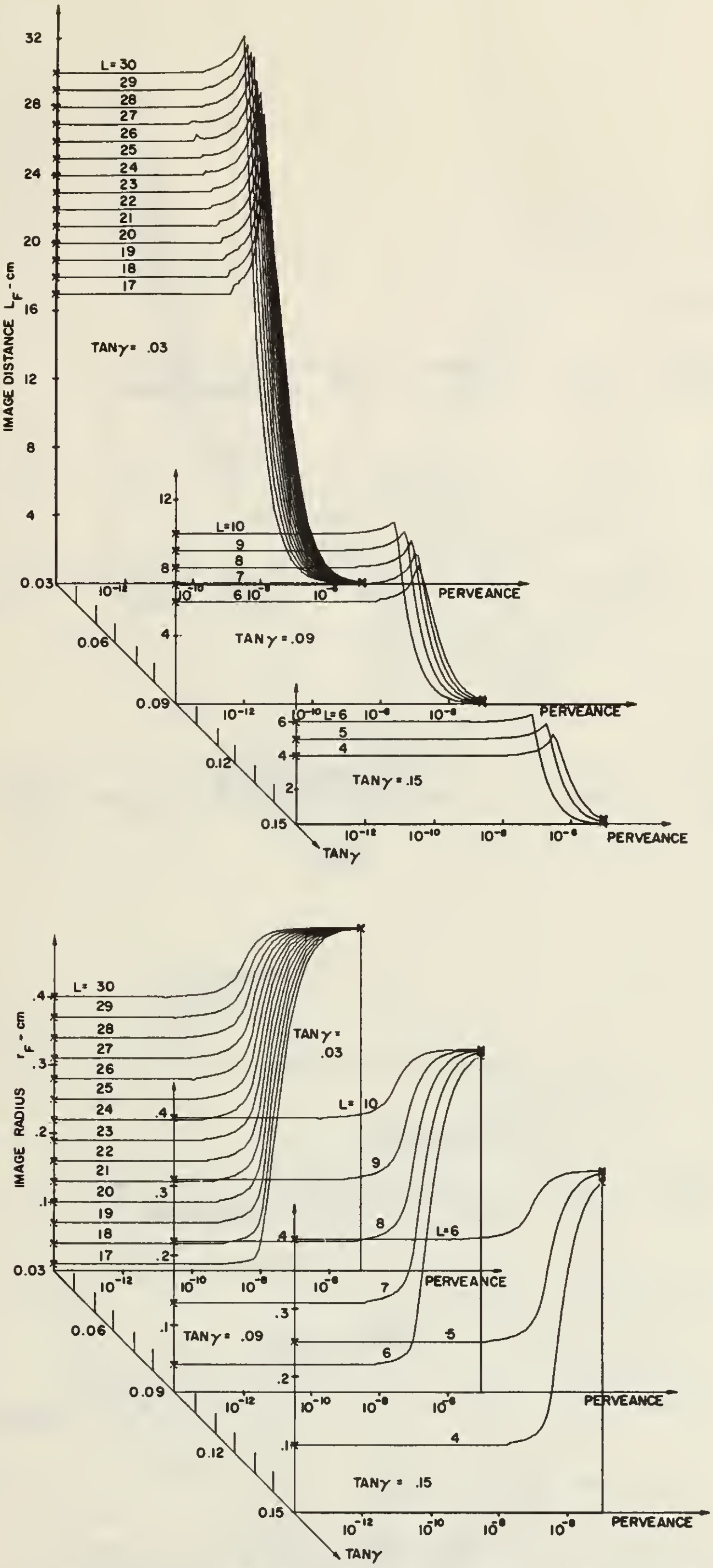


IMAGE DISTANCE L_F AND IMAGE RADIUS r_F FOR ELECTRON BEAM (R_B = 0.50 cm)

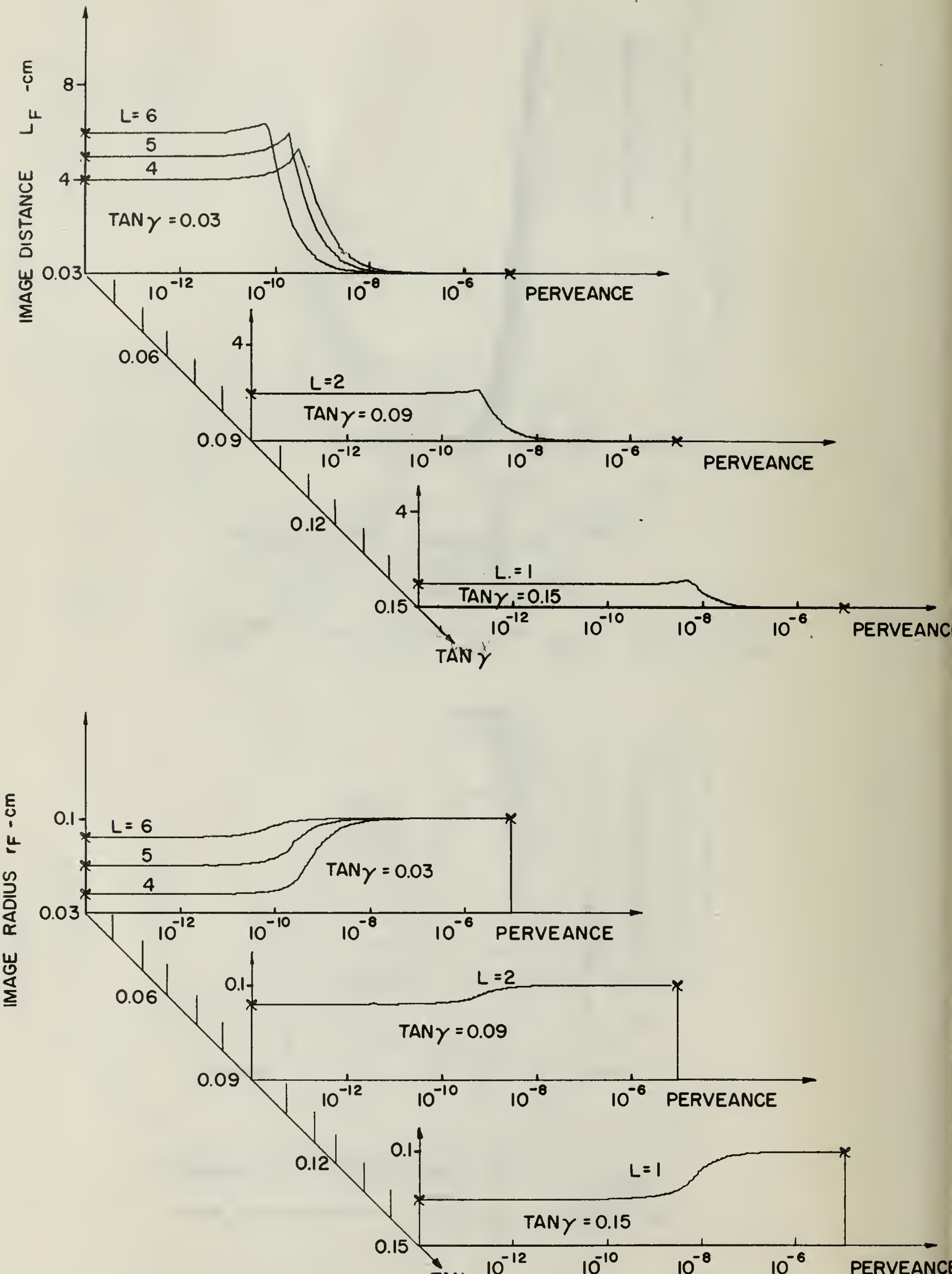


IMAGE DISTANCE L_F AND IMAGE RADIUS r_F FOR $^1H^+$ BEAM (R_B = 0.10cm)

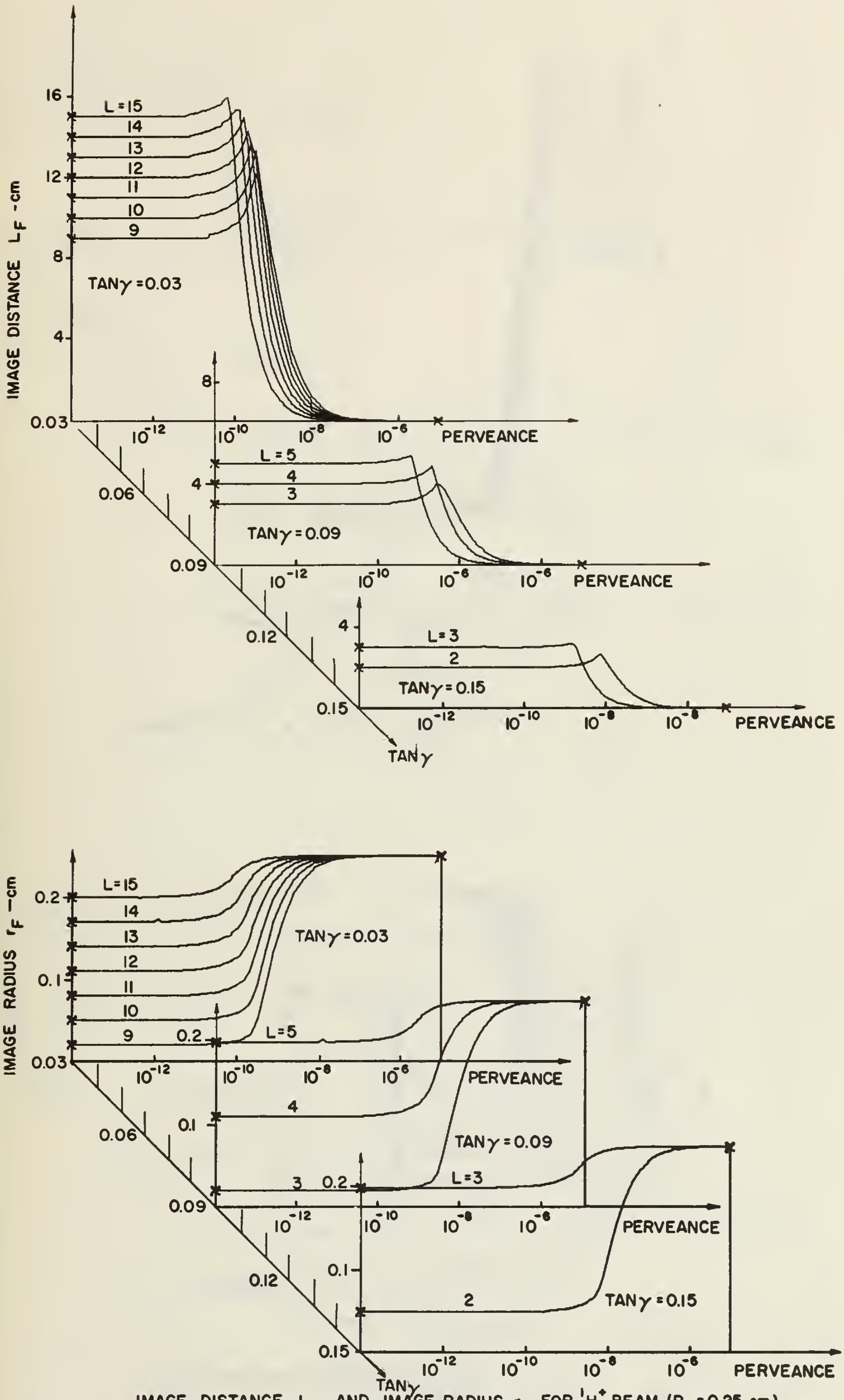


IMAGE DISTANCE L_F AND IMAGE RADIUS r_F FOR $^1H^+$ BEAM (R_B = 0.25 cm)

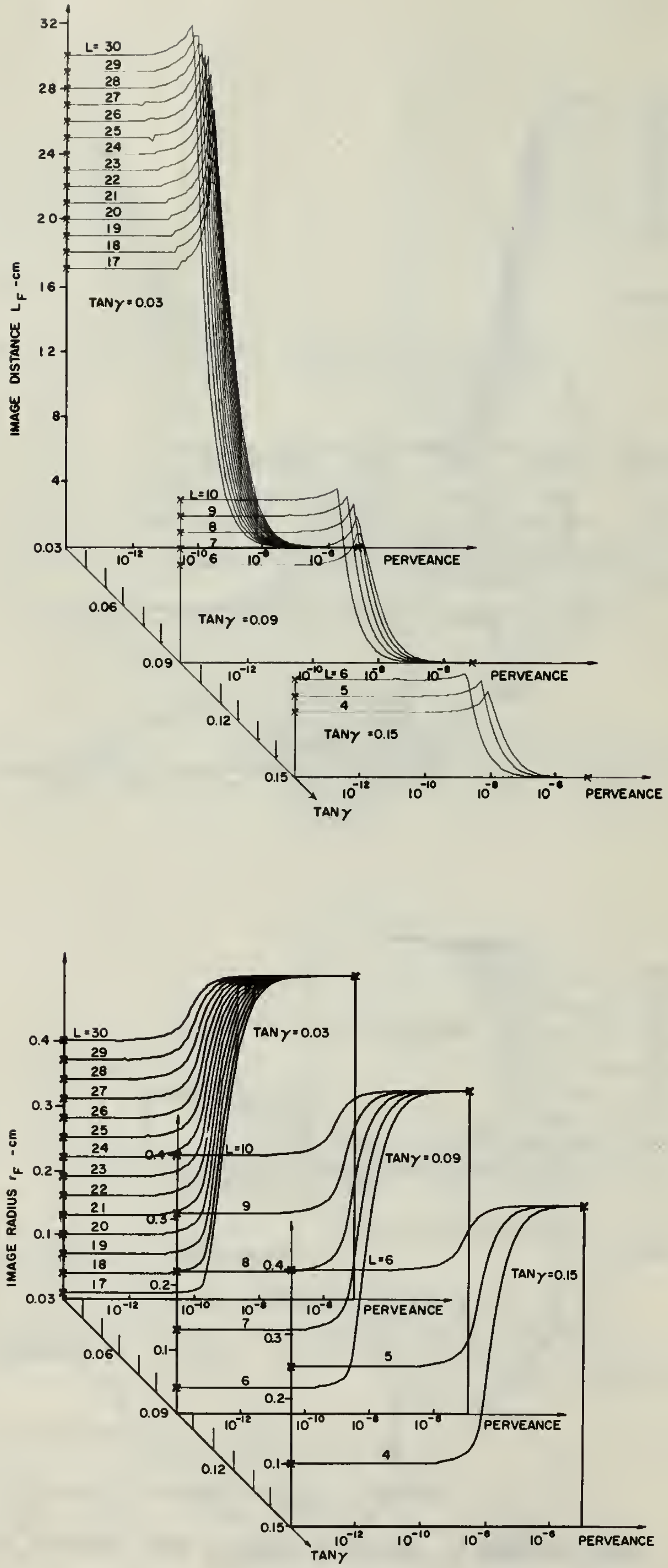


IMAGE DISTANCE L_F AND IMAGE RADIUS r_F FOR $^1H^+$ BEAM (R_B = 0.50cm)

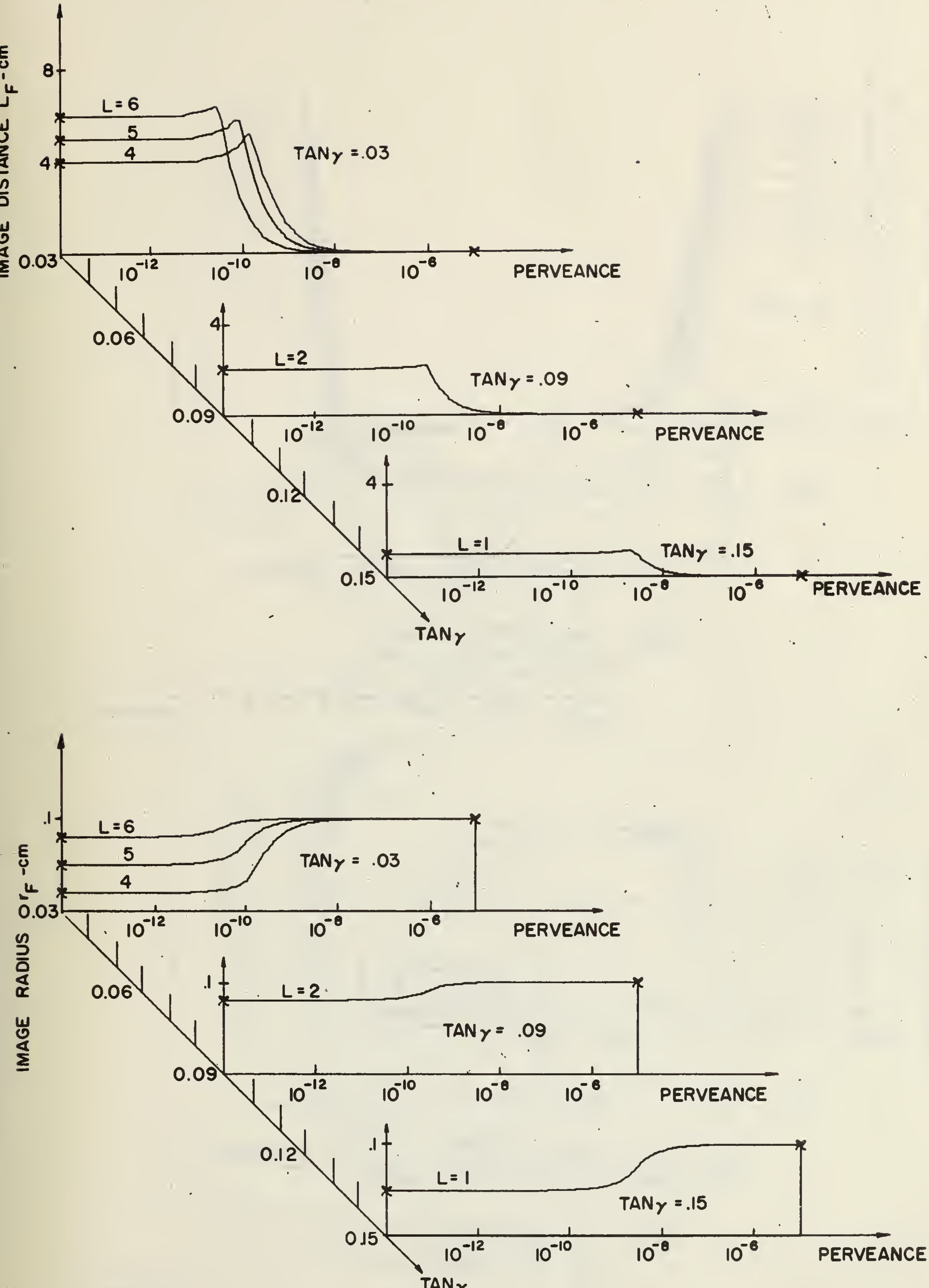


IMAGE DISTANCE L_F AND IMAGE RADIUS r_F FOR $^7Li^+$ BEAM (R_B = .10 cm)

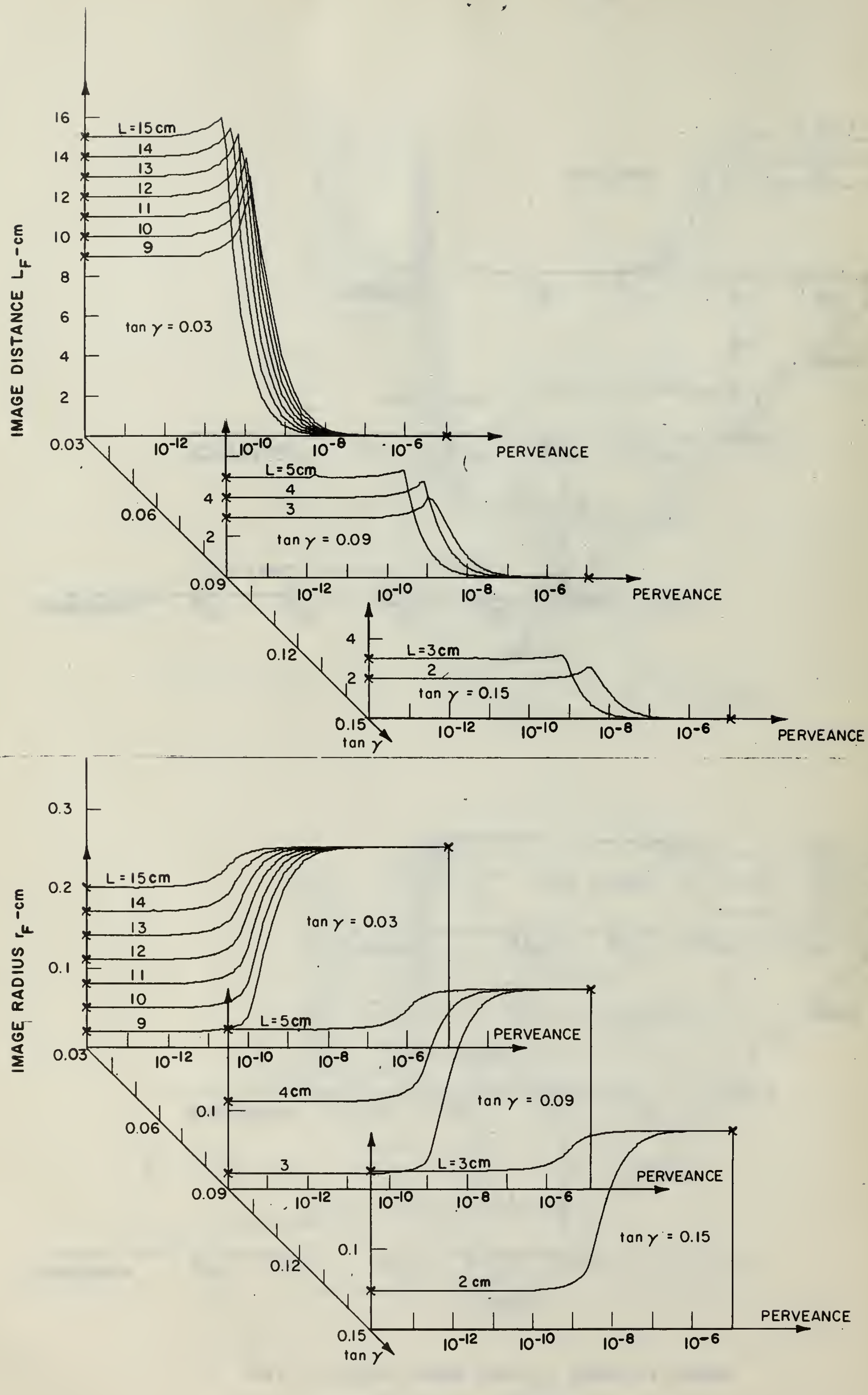


IMAGE DISTANCE L_F AND IMAGE RADIUS r_F FOR $^7Li^+$ BEAM (R_B = 0.25 cm.)

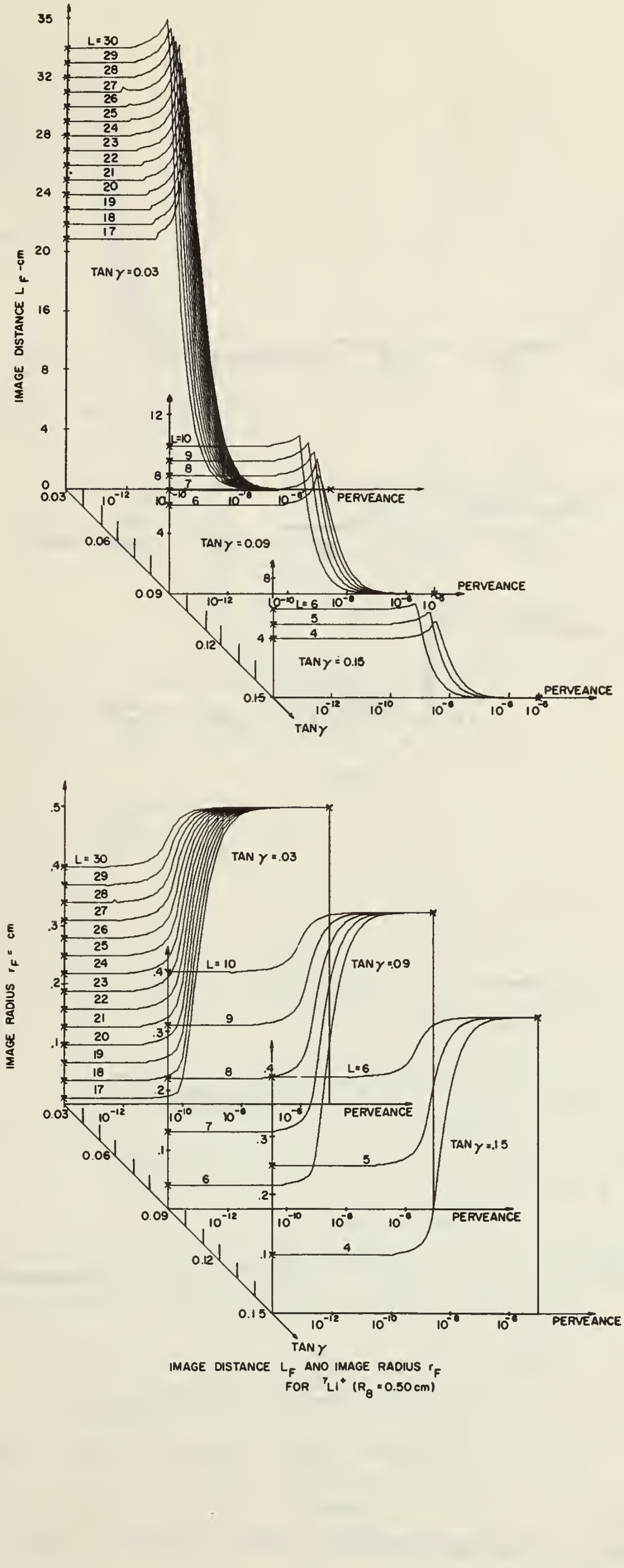


IMAGE DISTANCE L_F AND IMAGE RADIUS r_F
FOR $^7Li^+$ (R_B = 0.50 cm)

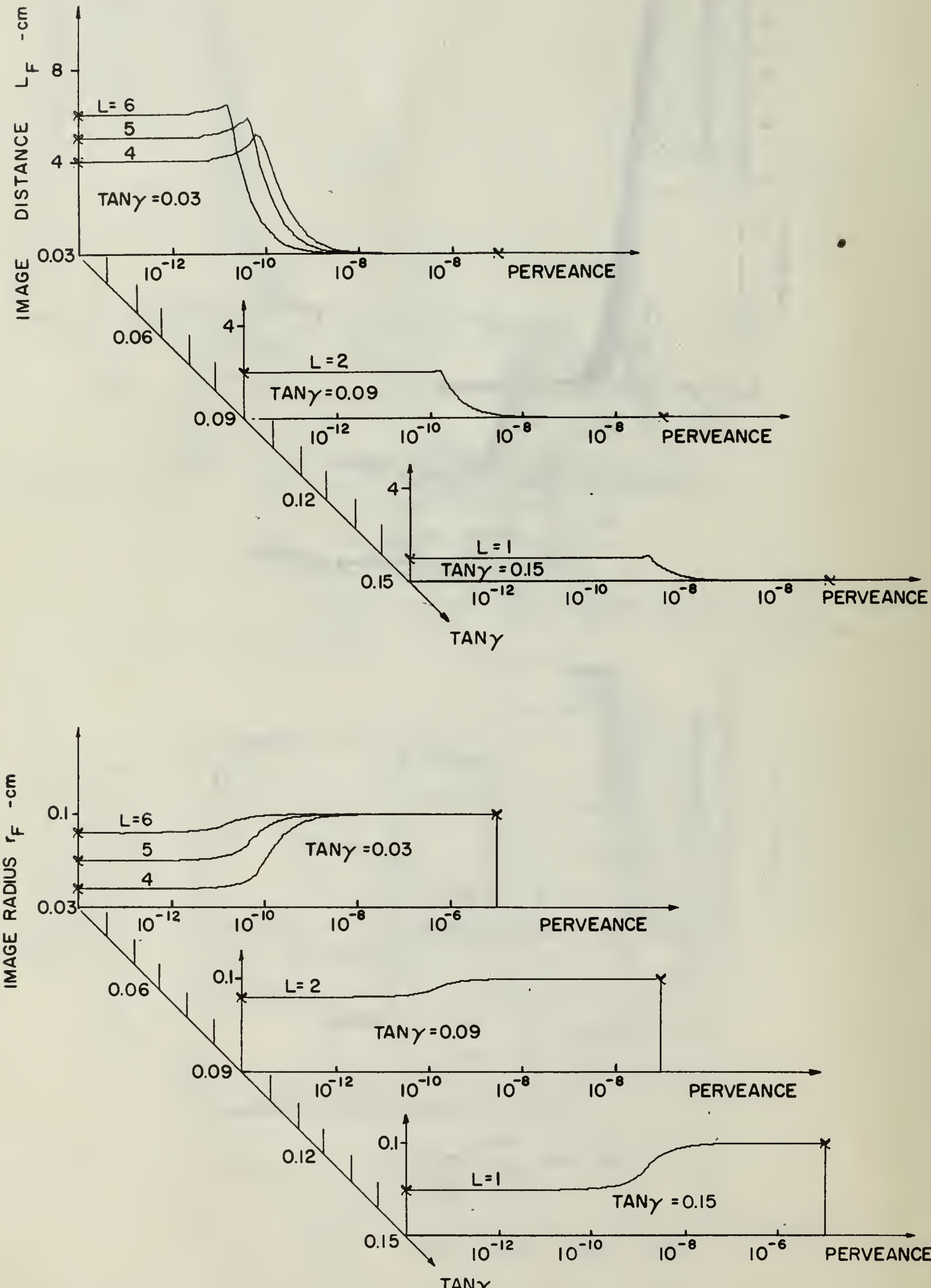


IMAGE DISTANCE L_F AND IMAGE RADIUS r_F FOR $^{20}Ne^+$ BEAM (R_B = 0.10 cm)

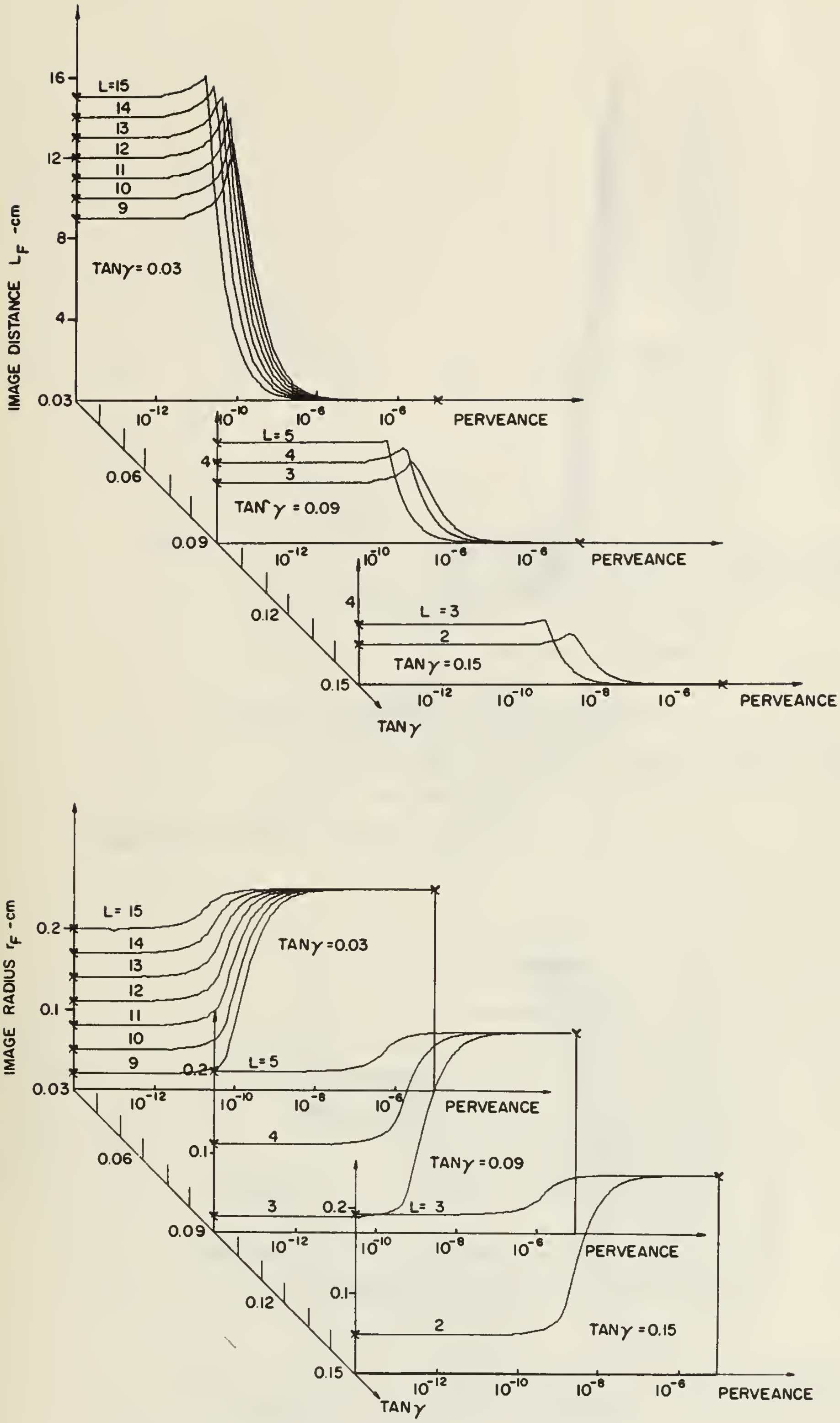
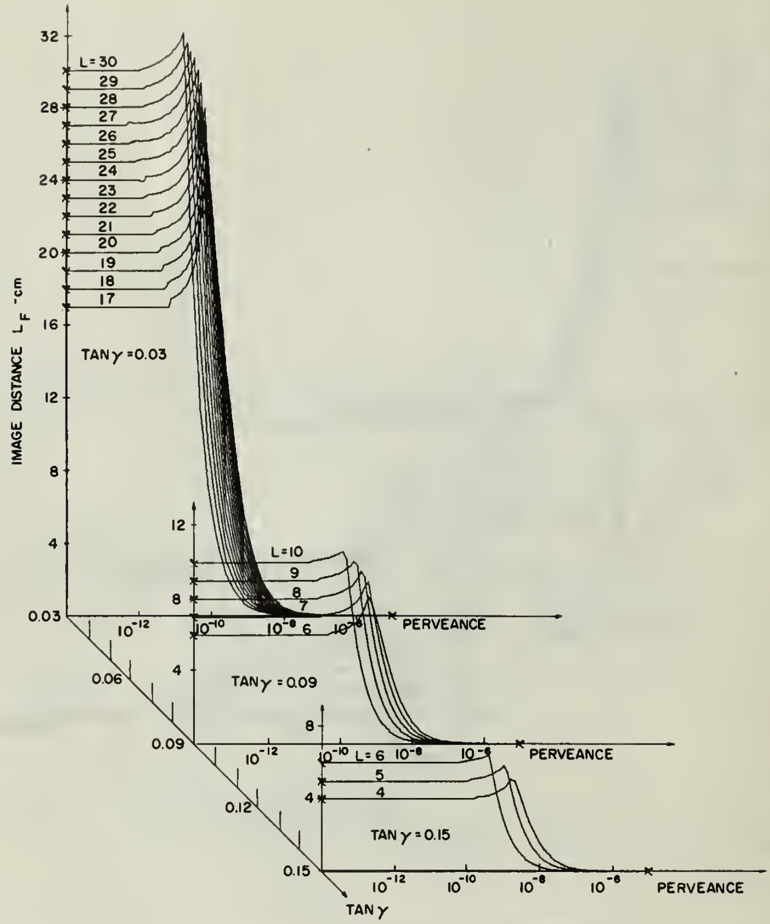


IMAGE DISTANCE L_F AND IMAGE RADIUS r_F FOR $^{20}Ne^{+}$ BEAM (R_B=0.25 cm)



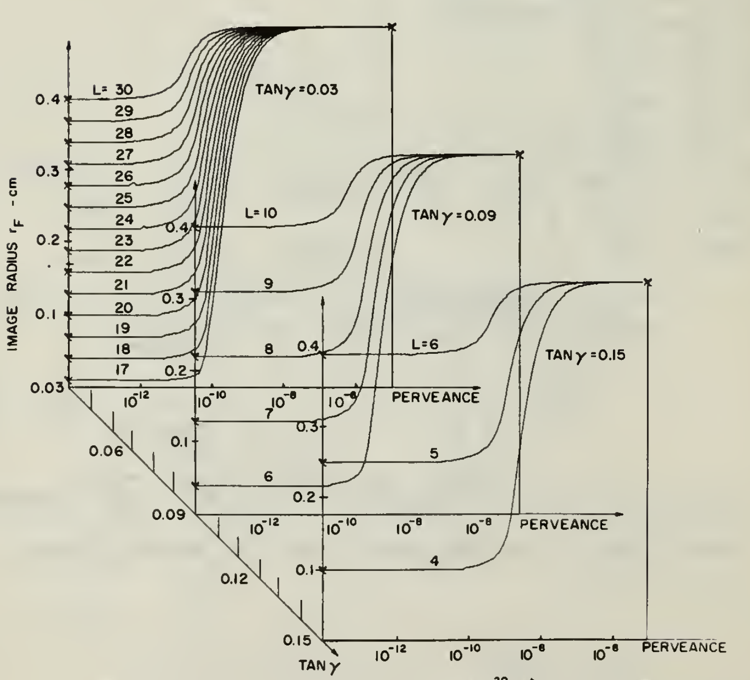


IMAGE DISTANCE L_F AND IMAGE RADIUS r_F FOR $^{20}Ne^{+}$ BEAM (R_B = 0.50 cm)

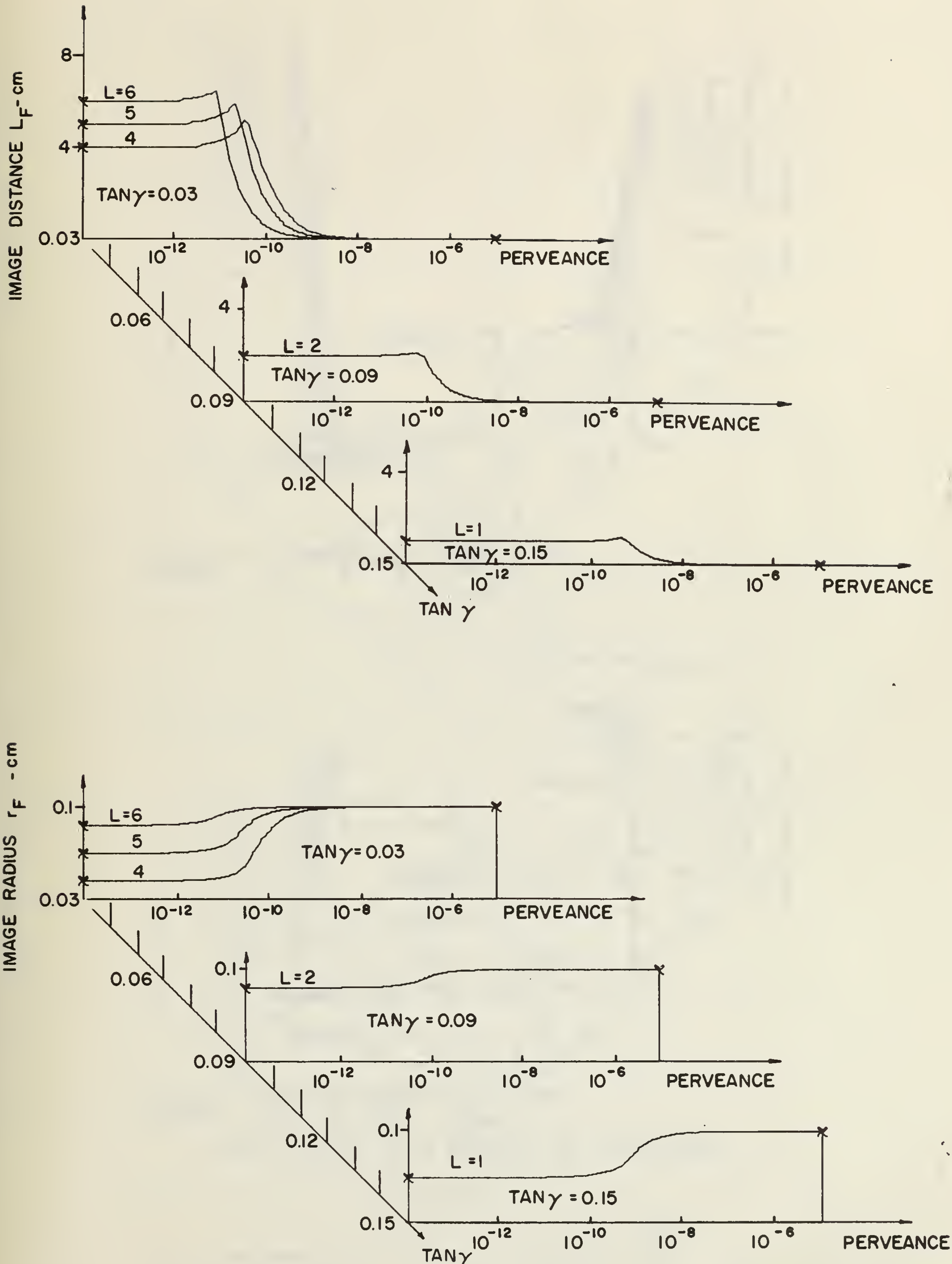


IMAGE DISTANCE L_F AND IMAGE RADIUS r_F FOR $^{85}Rb^+$ BEAM (R_B =0.10cm)

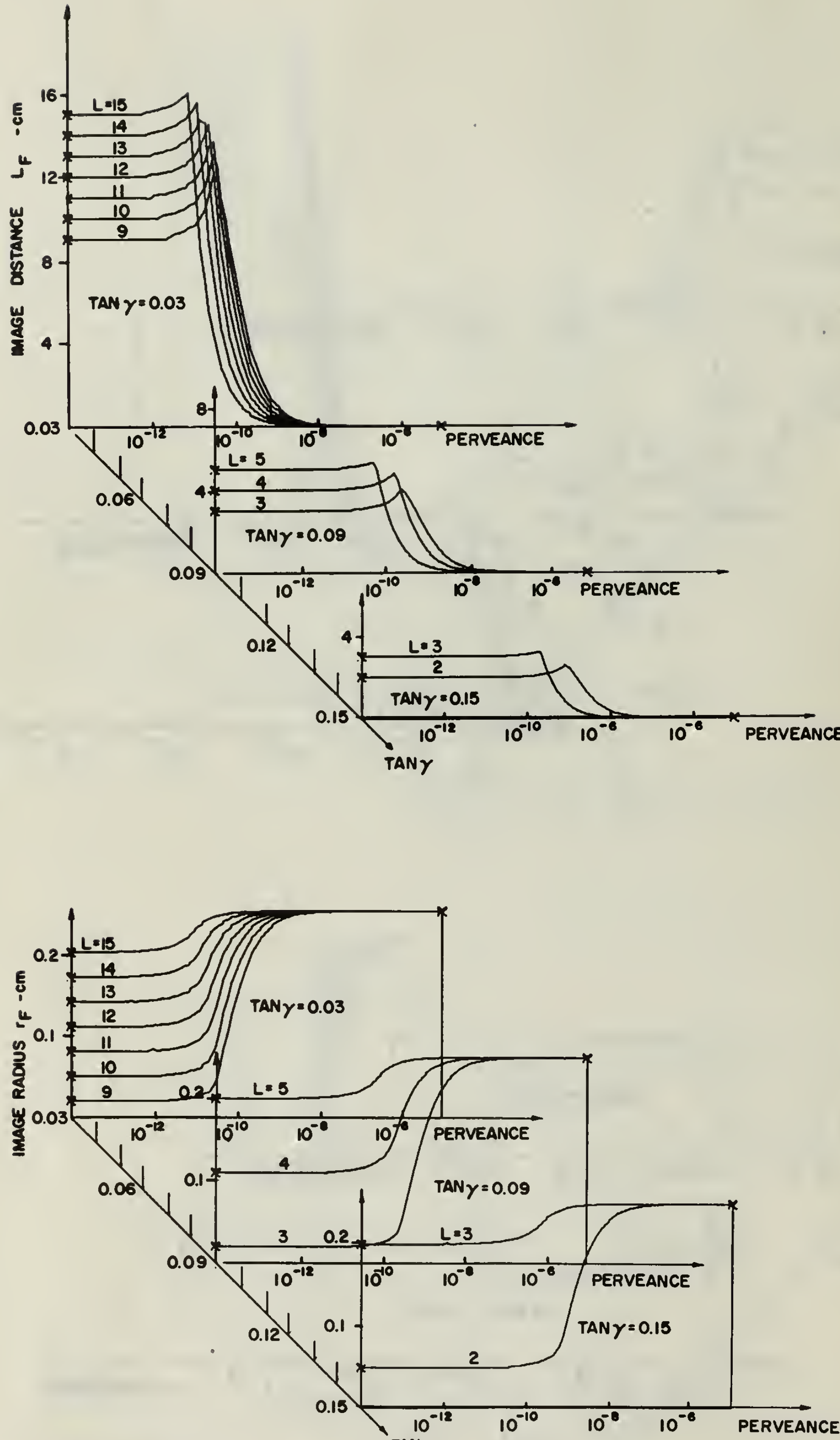


IMAGE DISTANCE L_F AND IMAGE RADIUS r_F FOR $^{85}RB^+$ BEAM (R_B = 0.25cm)

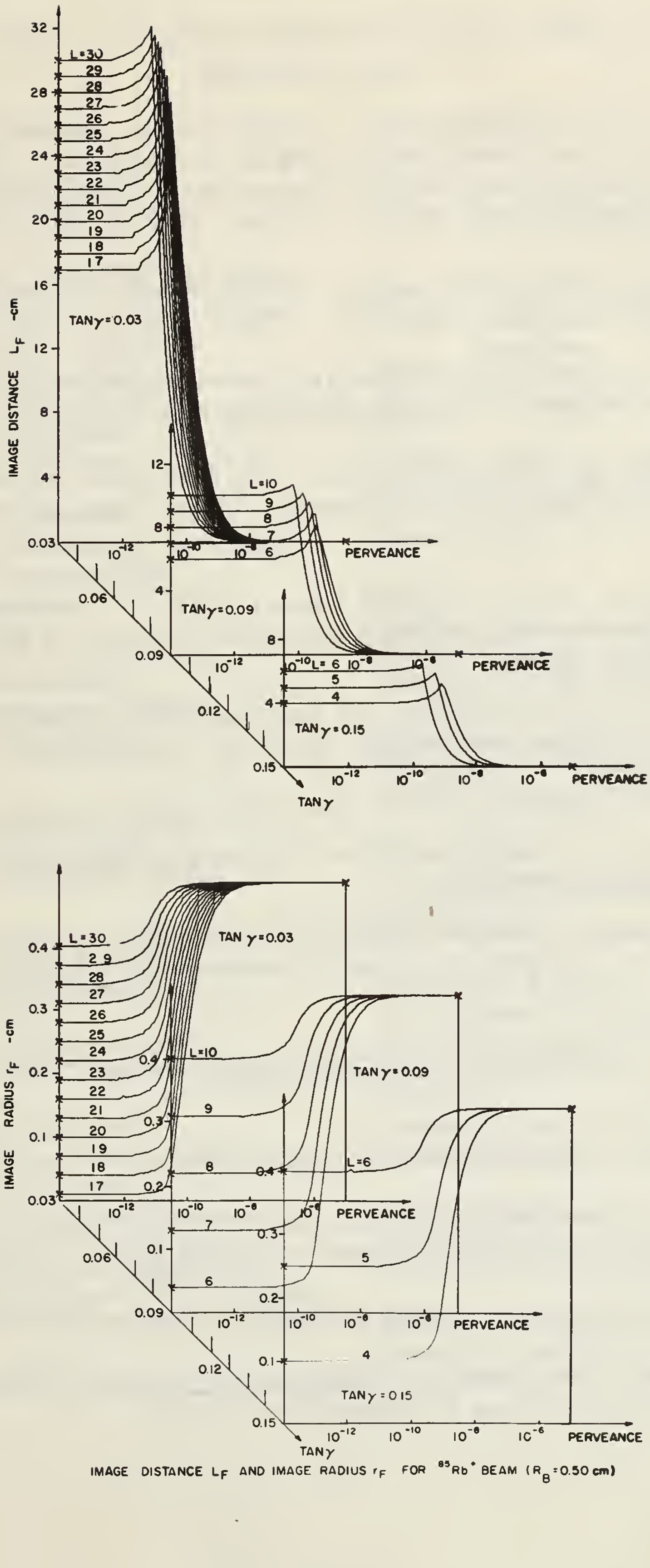


IMAGE DISTANCE L_F AND IMAGE RADIUS r_F FOR $^{85}Rb^{+}$ BEAM (R_B=0.50 cm)

IV. BIBLIOGRAPHY

1. Pierce, J.R., "Rectilinear Electron Flow in Beams", J. Appl. Physics 11, 548, 1940.

2. Simpson, J.A. and Kuyatt, C.E., "Limitations on Electron Beam Density of Unipotential Electron Guns at Low Voltages". Journal of Research of the National Bureau of Standards - C Engineering and Instrumentation Vol 67C, No. 4. Oct-Dec 1963, p. 279 - 281.

3. Simpson, J.A. and Kuyatt, C.E., "Design of Low Voltage Electron Guns," Rev. Sci. Instr., 34, 265, 1963.

4. Soa, E.A., "Systematische Untersuchungen an Elektrostatischen Inmersions Objectiven", Janaer Jahrbuch, 1, 115, 1959.

5. Spangenberg, K., and Field, L.M., "The Measured Characteristics of some Electrostatic Electron Lenses", Elect. Commun. 21, 194, 1943.

6. Haskell, H.B., Heinz, O., and Lorents, D.C., "A Multistage Gun for the Production of Low Energy Ion Beams,"Reviews of Scientific Instruments (in press)

   Haskell, H.B.," Investigation of a Low Energy Alkali Ion Gun," MS Thesis USN Postgraduate School, May 1965 (Unpublished).

7. Allison, S.K. and Kamegai, M., Rev. Sci. Instr. 32, 1090 (1961).

   Johnson, F.M., "Studies of the Ion Emitter Beta Eucryptite", RCA Review 22, 427 (1962).

   Lagerstrom, R.P., "Ion and Plasma Beam Sources", Quarterly Research Review No. 4, Stanford Electronics Laboratories, 1 Jan to 31 Mar 1963, pp 1 - 33 ff.

8. Glaser, W., Grundlagen der Elektronenoptik, Springer-Verlag (1952) p 69 ff.

9. Watson, E.E., "Dispersion of an Electron Beam," Phil Mag 3, 849 (1927).

10. Wendt, G., "The Spreading of an Electron Optical Pencil Under the Influence of its own Charge", Ann. d. Physik 2, 256 (1948).

11. Jahnke, E. & Emde, F., Tables of Functions, Dover Publications (1945).

12. Morse, P.M. & Feshbach, H., Methods of Theoretical Physics, McGraw-Hill (1953) Part I, Chapter 5, p. 523 ff.

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